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AGENT-BASED COOPERATIVE CONTROL

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14. ABSTRACT
The report addresses the formulation of a general theoretical framework for issues unique to cooperative control. The approach taken is to unify the fundamental principles of control Lyapunov functions, potential field theory, and the so-called optimal return function. These three principles are woven together to achieve an analytically vigorous formulation that addresses the required functionality of cooperative control problems. The development is prepared in the context of a multiple UAV cooperative ground moving target engagement scenario.

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AGENT-BASED COOPERATIVE CONTROL

1. INTRODUCTION

This research has sought to formulate a general theoretical framework for the cooperative control of multiple dynamic systems or units. There are two classes of cooperative control problems: those concerning manipulators and those concerning mobile robots; this proposed work is with regard to the latter. The most straightforward examples of such systems are groups or teams of autonomous vehicles cooperating to achieve a common goal. The "cooperative" aspect of the problem implies that no benefit is derived by the team if only a single vehicle performs a task, and that a high degree of *tightly coupled, coordinated action* is required. The class of problems considered generically in this proposal therefore requires the simultaneous action of multiple vehicles in a coordinated fashion.

Interest in cooperative control of mobile robots has been heightened in recent years mainly due to future planned scenarios involving multiple Unmanned Aerial Vehicles (UAVs) for the United States Air Force. Despite the heightened research activity in this area, *significant* issues with regard to both theory and implementation still exist. Theoretically, much work has been done regarding different aspects of the problem, but few if any attempts have been made to formulate a theoretical framework that would unify these somewhat separate efforts. The question then arises, "Why is establishing a theoretical framework worth it? What does it buy you?"

> ***The overarching philosophy behind this work is the following: Significant implementation issues exist with regard to the cooperative control of multiple autonomous vehicles. Establishing a theoretical framework that is capable of universally and analytically treating low, mid and high level planning and control is a necessary step toward addressing such implementation issues. The developed framework will allow questions regarding, but not limited to, communicated information requirements, robustness to communication delays and asynchronous communication, input saturation constraints, and non-holonomic behavior, to be posed and addressed mathematically.***

The proposed work is rooted firmly in dynamical systems and model-based control theory in an effort to addresses issues that have not previously received much attention – namely stability and robustness within a well-founded continuous-time analytical control structure. This is somewhat of a departure from the vast amount of work currently being done in cooperative control from a computational and network communications protocol point-of-view. Continuation of this work will help determine the minimum requirements for these difficult technological and system architecture issues by providing an efficient, well-founded structure to control such systems.

As will be shown for a number of applications, it will be useful and desirable to achieve the necessary coordination or cooperation without the use of a central controller/planner. In the language of control, this amounts to *not* casting the problem as a multivariable control problem where the control law is computed with full state information of all the vehicles under consideration. Therefore, the challenge is to define the interaction among the vehicles, or units, in a non-centralized way that divides the task between either identical (homogeneous), or specialized (heterogeneous) units, such that each unit can exercise a local control law based on local information (the unit's own state, and information exchanged between units) that contributes toward achieving a global goal (minimizing a team-wide cost). In this manner, the units will serve as "agents" of a team policy that is cooperative and distributed. ***The "agent-based" approach proposed therefore seeks to formulate the cooperative control problem such that local control laws can be exercised by the units, while still enforcing an analytically guaranteed team policy with all the assurances of stability, robustness and performance normally sought in the field of control theory.***

Fundamentally, this research seeks to help answer the following general open-ended questions regarding the theoretical challenges presented by cooperative control:

- Can a cohesive analytical framework be developed for the control of multiple dynamic systems that provides analytical assurances of stability, robustness, convergence, and other quantifiable metrics, regarding both the team and the individual units?
- What is an appropriate set of principles upon which to formulate this framework? Can an ad-hoc set of rules be avoided in favor of fundamental principles?
- If so, will a framework grounded in fundamental principles reveal a minimalist structure regarding the required information exchange and communication among the dynamic units?
- If so, can the overall problem be defined in a manner that is general enough so that the resulting framework applies to a broad class of problems?

To help answer these and other related questions, a framework will be developed such that well established analysis tools can be utilized to show robust and stable control on both the local and global level. In brief, the **core idea** of this proposed framework will be to interpret the following three fundamental principles in a unifying manner within the requirements of the class of cooperative control problems under consideration: 1. control Lyapunov functions, 2. potential field theory, 3. the optimal return or value function from dynamic programming. This core idea follows from an assumption that stable, robust and effective cooperative control must follow from, and include, considerations of stability and robustness of each individual unit. So the reader may now appropriately ask, “So what’s new here, and how is this not just three ideas in a blender? What will be the real contribution?” ***This bottom-up approach is not hierarchical, but rather it is unified in nature. It will unveil the amount of information exchange required, remove the ambiguity surrounding closing control loops at various levels, and provide analytical assurances of stability, robustness and performance.***

THE CORE IDEA: Unification of three principles to form a theoretical framework for cooperative control:

Control Lyapunov Function	**Potential Field**	**Optimal Return Function**
Addresses: issues of stability and robustness through model-based nonlinear control theory.	Addresses: issues of desired interaction such as coordination, formation, and collision avoidance.	Addresses: issues regarding assignment – what task among several the team performs, and which nearby vehicles participate.

The unification of these three principles will guide a well-founded and fundamental inclusion of the following considerations:

- Individual vehicle dynamics (including non-holonomic dynamics and constraints)
- Minimal required inter-vehicle communication and information exchange
- Low-level control for path following and trajectory tracking
- Collision avoidance with environmental obstacles and other vehicles
- Cooperative precision configuration/formation and pursuit/evasion/search team control
- Vehicle assignment (forming teams from available vehicles for lowest cost completion)

The core idea will allow all of the above considerations to be cast within a single unified structure so that a rigorous analysis of stability, performance and robustness of the entire system of cooperating units can be made. Put another way: the core idea will allow all of these operational considerations to be viewed in the same fundamental manner. Low-level control, desired real-time trajectory generation, vehicle cooperation and interaction, and task assignment will all be able to be considered simultaneously.

2. LITERATURE REVIEW

In reviewing the scientific literature, there are four key areas that have made contributions relevant to the cooperative control problem class. These areas are: artificial potential fields for obstacle avoidance, motion planning for nonholonomic vehicles, swarm formation control, and the optimal assignment of vehicles to tasks. Although the work in these areas has been quite significant and extensive, the *integration* of these contributions within the context of cooperative control is not mature. Establishing a framework for cooperative control will bind together these four disparate areas of research. Although some important exceptions can be found, artificial potentials have *typically* not been applied to cooperative problems, swarm formation control is typically unstructured and not highly coordinated, motion planning for nonholonomic vehicles typically involves only a single vehicle, and optimal assignment is typically performed open-loop as a static optimization problem and not cast in continuous-time. The proposed framework will tie together these areas under one umbrella to enable the design and analysis of cooperative control systems utilizing either homogenous or heterogeneous vehicles.

Khatib [40,41,42] pioneered the use of artificial potential fields for obstacle avoidance of manipulators and mobile robots. Goal points were set as attractors and obstacles were set as repulsive, and the desired path was the resulting gradient of this combined field. The gradient was tracked using feedback linearization. A recognized drawback of this initial approach was that exact knowledge of the dynamics were required to ensure stability (in other words the method was not robust). Koditschek [45,46] modified this approach by applying energetic arguments to provide robustness to a class of dissipative mechanical systems. Newman and Hogan [70] built upon the work of Khatib for obstacle avoidance for robot manipulators. They explicitly built joint torque constraints into the potential field by defining a time varying potential based on energetic arguments. Rimon and Koditschek [90] proposed a method of constructing potential field-based navigation functions for robot navigation around obstacles that result in a bounded torque feedback controller. Guldner and Utkin [32,33] proposed a potential based method for obstacle avoidance by a holonomic robot. The key feature of the method was that the control law is not based directly on the gradient of the potential field, but rather the gradient lines were interpreted as the desired trajectories. A sliding mode approach is then adopted to converge to a sliding surface that takes the robot to the gradient line and provides exact tracking once on the gradient line. This modification to Khatib's early approach of deriving the control law directly from the potential field provides robustness to modeling errors. In this manner, the work of Guldner and Utkin forms a link between a control Lyapunov function and a potential field. Furthermore, it establishes a method that decouples the design of the potential field from the design of a controller to track gradient lines. The method however is presented for a single robot and with non-time varying potential fields; how to extend this work to include interaction between two or more robots with their own potentials plus an environmental potential is unclear.

The early work of Dubins [22] established interest in the static optimization problem of trajectory or path planning for a single non-holonomic vehicle with minimum turning radius constraints. Murray [67] and Tilbury [99] (sinusoids), Leonard [56], Bloch [13,14,15], and Nair [69] (controlled Lagrangians), Nair [68] and Åström [2] (energy approaches), and others [52,104] have also tackled various aspects of the constrained input problem. With regard to real-time path planning, Primbs et al [81,82,26] offer some very keen observations regarding the connections between the value function of the Hamilton-Jacobi-Bellman equation, control Lyapunov functions (CLF), and minimum norm controllers through Sontag's formula. This work begins to make some of the desired connections between CLF's, potential fields and the optimal return function sought by this proposed work. Essentially, the paper points out that the gradient of a CLF is simply some scaling of the gradient of the value function (or optimal return function) from the control law solution of the HJB equation, $V_{\mathbf{x}}^{*} = \lambda(\mathbf{x})V_{\mathbf{x}}$. It can also be shown that a given CLF, *V*, corresponds to the value function, V^{*}, for some set of cost functions (the so-called "inverse optimal" property). The dynamic window approach to obstacle avoidance in Ogren and Leonard [72] uses the connections pointed out by

Primbs to beautifully merge aspects of model predictive control and control Lyapunov functions to provide a theoretical assurance of convergence for the navigation of a single mobile robot. Essentially, the result is a potential field method, through the construction of a navigation function, with assurances of convergence. The method neatly addresses the concern that arises when simply utilizing the gradient lines of the potential field as desired trajectories thereby making them state dependent and in turn no longer exogenous inputs and hence spoiling the stability and robustness of using a Lyapunov method to track them. Instead, the navigation function (potential function) is incorporated into the CLF directly along with an additional more traditional tracking error term: $V(r,\dot{r}) = \frac{1}{2}\dot{r}^T\dot{r} + NF(r)$, $\dot{V} = \dot{r}^T u + \dot{r}^T \nabla NF(r) \le -\varepsilon\|\dot{r}\|$ where the plant is given by $\ddot{r} = u$. Although well prescribed, the construction of the navigation function is performed via solving the shortest-path problem cell-wise on an obstacle grid map, and hence must be computed ahead of time (i.e. not in real-time). The method also leaves questions regarding establishing a connection between the CLF and the optimal return function. Within the realm of *cooperative* control of mobile robots, work has been done on formation and swarm control utilizing potential field ideas [t35,t50,t57], employing stability arguments [58,27,65,97], graph theory [21,74], invariant manifold techniques [78], decentralized approaches [54,62,55], leader-follower approaches [48], platooning [31,103,36,84,95], and hybrid and multi-modal control approaches [25,106,50,49].

The assignment of multiple available vehicles to various tasks has been approached by many including Passino [80], Beard [5,6], Liu [60], Chandler [17], Schumacher [93], Pachter [76], and others [66,83,63,71], but typically the problem is approached from a non-dynamical systems point of view. Cooperative path planning via mixed integer linear programming is a popular non-dynamical systems approach [89,94]. Various other approaches begin to make some connections between an optimal return type function for decisions and a team metric akin to a CLF, for assignment type problems: satisficing approaches [85,77,24,20,38], Lyapunov functionals [35], Lyapunov certificates [101], probability of loss [9], behavior-based approaches [92,3], consensus approaches [75,88], biologically inspired strategies [29,59], and auction methods [30]. The unification of vehicle dynamics and real-time cooperative path planning and assignment remains a difficult challenge.

Although significant in their achievements, there has been no effort among prior works to unify all of the above concerns (namely: model-based control theory, cooperative interaction, real-time path planning, and assignment) into a single cohesive theoretical framework.

3. Proposed Control Framework

As stated in the introduction, the objective of the proposed research is to formulate a general theoretical framework to address the issues unique to cooperative control. The approach to be taken is to unify the fundamental principles of control Lyapunov functions, potential field theory, and the so-called optimal return function. The flowchart in Figure 1 shows how these three principles will be woven together (left-hand column) to achieve an analytically rigorous formulation that addresses the required functionality presented by cooperative control problems (shown in the right hand column).

The framework consists of four principle elements: the optimal return function (ORF), the cooperative control Lyapunov function (CCLF), the Potential Function (PF), and the agent control function (ACF). The CCLF ensures that the team asymptotically reduces the team cost. The time derivative of the CCLF is split among the agents with an associated ACF being assigned to each agent, whereby this step defines an agent and its role. The ACF's incorporate PF's to dictate local agent-to-agent interaction (e.g. collision avoidance), and to accommodate dynamic and input constraints (e.g. nonholonomic and control saturation). Effectively the PF specifies a portion of each agent's desired trajectory, while the other portion comes from the CCLF. If the time derivative of each ACF can be selected negative definite, then the CCLF will be negative definite in a sufficient sense implying the team and each agent will be assured to be asymptotically stable. If the time derivative of every ACF is not selected negative definite, but

rather in a way such that all ACF's *combine* to form a negative definite time derivative of the CCLF, then the team is ensured asymptotically stable but each agent is only ensured to be bounded-input bounded-output stable. This second case will give the team the ability to make individual sacrifices (non-optimal for an individual agent) for the sake of benefiting the team. Finally, the ORF represents the optimal "cost-to-go" and will be related analytically to the CCLF. If the CCLF is viewed as a generalized distance to the goal, and the time derivative of the CCLF (the generalized velocity toward the goal) is designed to be a particular analytical function, then the generalized time to reach the goal can be solved in closed form. The ORF is therefore the optimal return function for a particular team task. Various team tasks and their associated ORF's can then be evaluated in order to select the optimal task for the team from among a discrete set of possible team tasks (optimal in the sense of the ORF plus the cost to switch tasks). Various combinations of different vehicles available to be assigned to a task (the assignment problem) can then be selected by evaluating ORF's for each combination.

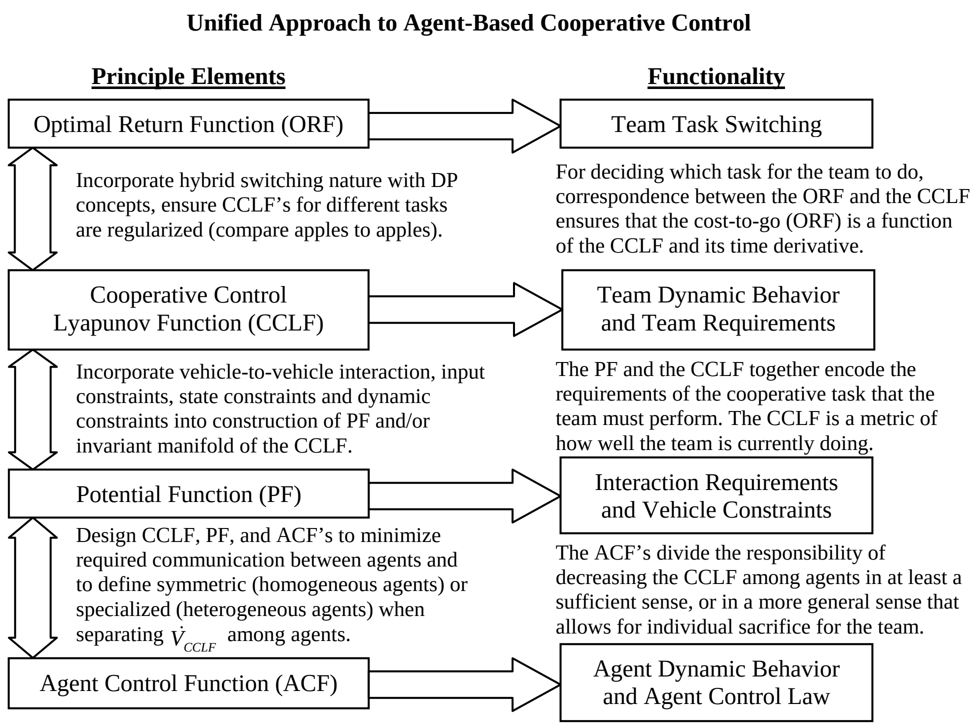

Figure 1: Flowchart of the main principles, connections between them, and resulting functionality in the proposed agent-based cooperative control framework. The framework is unified in the sense that the global convergence, stability, and robustness is ensured by virtue of the structure of the framework.

3.1 Scientific Relevance

The proposed development of a framework for cooperative control seeks to add an analytical structure to a relatively young research topic with applications extremely important to the nation. By seeking to base this framework in fundamental principles, ad-hoc and heuristic approaches will be able to be replaced with methods that will provide solid analysis and well understood control design. Further, by combining the as yet disparate areas of artificial potential fields, swarm formation control, motion planning for nonholonomic vehicles, and optimal assignment of vehicles to tasks, with a unique unifying interpretation, this work will help lay the groundwork for an emerging branch of controls – cooperative control.

3.2 Relevance to the Air Force

In addition to formulating a generic approach that applies to a large class of problems under the umbrella of cooperative control, the proposed research program will also address a number of specific applications. A currently existing platform awaiting future conceived cooperative Unmanned Aerial Vehicle (UAV) missions, is the UCAVs pictured in Figure 2. Below is a list of military (see attached letter of support) and non-military cooperative control applications which are planned to be *specifically addressed* by the developed framework. Selected applications in bold below with be formulated and simulated as a simplified planar problem, experimentally verified with non-holonomic wheeled platforms in the lab, and then extended to the 6-DOF, 3-D version in collaboration with the Air Force and its extensive simulation software (non-classified and available for university use).

Figure 2: Boeing X-45A Uninhabited Combat Aerial Vehicles (UCAV). A pair on the left, and flying alongside a manned F-16 on the right.

Military Applications

In the **CGMTE (Cooperative Ground Moving Target Engagement) s**cenario, two or more UAV's must circle a ground target such that their sensor footprints overlap in a particular configuration to provide adequate fused accuracy to localize the target. The cooperative control laws enacted must fly the UAV's to maintain this sensor fusion requirement, while accommodating non-holonomic dynamics, turning rate constraints, and avoiding collisions. In the **Cooperative Visual Target Tracking** scenario, it is envisioned that multiple UAV's will track targets visually. Imagine a single UAV visually locating a ground target moving among non-targets. In order to track the target with additional UAV's, each additional UAV must first acquire a visual from a similar perspective and range. This hand-off will require UAV's to cooperatively fly very near each other. The similar problems of **in-flight refueling with autonomous vehicles** (a heterogeneous problem) and **cooperative laser designation** will also be pursued. The so-called **SEAD (Suppression of Enemy Air Defenses) Missions** destroy, deceive or jam an enemy's Integrated Air Defense Systems (IADS). IADS typically consist of a number of ground-based tracking radar stations (that compare data in real-time to enhance their detection abilities) and a number of surface-to-air missile (SAM) launch sites. To **destroy an IADS**, it is envisioned that one or more UAV's will jam appropriate radar stations while another masked aircraft bombs the SAMs. To **deceive an IADS**, multiple UAV's must cooperatively present a phantom aircraft radar image by sending delayed radar pulses to specific radar stations (the UAV's are assumed radar invisible). If the phantom image is not adequately correlated, the IADS will detect that it is a deception. To **cooperatively jam an IADS**, multiple UAV's must shrink the detection range of radar stations cooperatively in order to route a non-radar-invisible aircraft through the IADS region. Finally the **Convoy Escort** problem is one where a convoy of ground vehicles must be protected by monitoring for enemies on its flanks and ahead of the convoy. Given that UAV's fly much faster than the ground vehicles being escorted, they must cooperatively switch off monitoring one of the two sides, or the front, in a manner such that there are minimal gaps in the monitored regions.

Non-Military Applications

Listed below are also a number of closely related but non-military autonomous cooperative scenarios imagined for mobile robots. In **coordinated search and rescue** it envisioned that multiple autonomous ground vehicles or aerial vehicles will cooperatively search for victims of such situations as fallen buildings or sea search and rescue. This application presents the opportunity to combine potential field ideas and probability density functions to achieve effective searching. **Cooperative forest and brush fire suppression** presents similar theoretical development opportunities. Cooperative monitoring applications

such as **border patrol** and **atmospheric profiling** mirror the motivations of the military convoy escort problem above. Problems associated with **intelligent highway systems** also represent the type of tightly coupled coordination sought to be addressed by the proposed framework.

4. TECHNICAL APPROACH - INITIAL RESULTS AND DISCUSSION

This section provides a preliminary investigation done to show the viability of the proposed framework. Specifically, this preliminary work demonstrates connections between the notion of potential fields and control Lyapunov functions. It also shows the relationship between the CCLF and the formation of the ACF's.

Figure 3 below shows a bit more detail regarding the connections between the ORF serving as an estimate for the cost of the team completing a task or satisfying and holding a certain configuration, the CCLF serving as a means of always reducing the total team cost, the PF serving as a means of incorporating interaction and vehicle constraints, and the ACF's serving to collectively decrement the CCLF under the constraints presented by the PF.

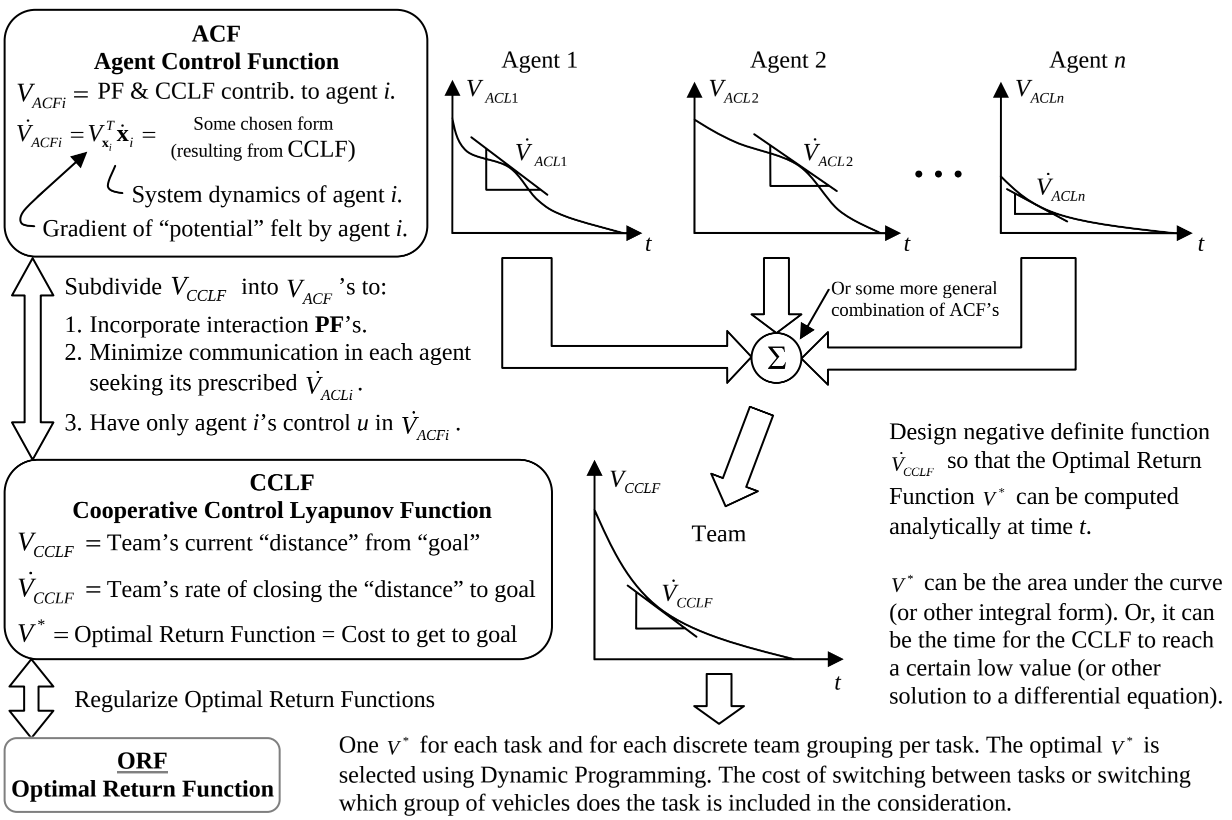

Figure 3: Connection diagram showing the unification of the principles of Lyapunov control functions, potential fields and the optimal return function.

4.1 Relating the Potential Field (PF), the Cooperative Control Lyapunov Function (CCLF) and the Agent Control Functions (ACF's)

This example will show how three components of the proposed framework begin to interrelate and fit together, namely the potential field (PF), the Cooperative Control Lyapunov Function (CCLF) and the Agent Control Function (ACF). Being preliminary work only, this example will of course not definitively resolve general issues regarding these three components of the approach, but rather it serves as a jumping-off point to begin to expose the critical issues that must be formally and theoretically addressed in pursuing the proposed framework.

In order to illustrate how the PF can be used to separate the cooperative control task into "agents" representing the multiple vehicles, and how each agent can be symmetric (i.e. look identical to other agents) and exercise a local control law (from the ACF) that contributes constructively along with other ACF's to minimizing the global cost (from the CCLF), consider the following example. Consider the simplified problem of two cooperating non-holonomic vehicles that must establish a particular orbit around a target with a particular angular separation between the two vehicles. This problem is considered as a simplified first step toward solving the Cooperative Ground Moving Target Engagement (CGMTE) problem. The scenario considered is shown in Figure 4 below.

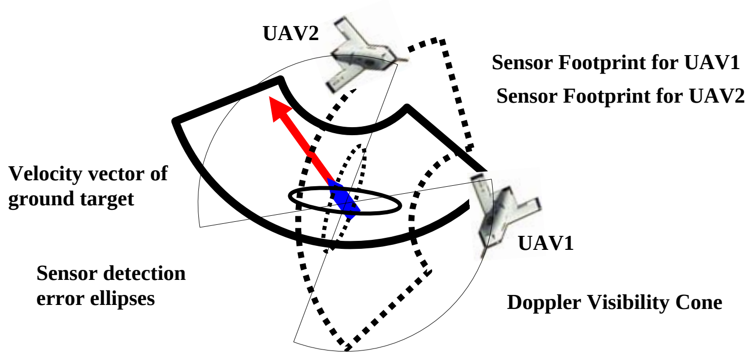

Figure 4: Cooperative Ground Moving Target Engagement (CGMTE) problem showing the cooperative requirements of two UAV's. The UAV's must be within the Doppler visibility cone, with their sensor footprints over the target, and with the error detection ellipses of each sensor contributing to a more precise and accurate localization of the target than achievable by either UAV individually.

Let us consider a two dimensional representation of the problem where the planar dynamics of each vehicle are given by the so-called Dubin's car model [22]:

$$\dot{x} = v\cos(\theta)\,,\ \ \dot{y} = v\sin(\theta)\,,\ \ \dot{\theta} = u \tag{1a}$$

where x and y are the Cartesian coordinates of the vehicle, θ is the heading angle, v is the velocity of the vehicle (assumed constant), and u is the control variable representing the heading angle rate of change. This can be represented more compactly as $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u$:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}}_{\dot{\mathbf{x}}}=\underbrace{\begin{bmatrix}v\cos(x_3)\\ v\sin(x_3)\\ 0\end{bmatrix}}_{\mathbf{f(x)}}+\underbrace{\begin{bmatrix}0\\ 0\\ 1\end{bmatrix}}_{\mathbf{b(x)}}u \tag{1b}$$

Two vehicles are then represented by $\dot{\mathbf{x}}_1=\mathbf{f}_1(\mathbf{x}_1)+\mathbf{b}_1(\mathbf{x}_1)u_1$ and $\dot{\mathbf{x}}_2=\mathbf{f}_2(\mathbf{x}_2)+\mathbf{b}_2(\mathbf{x}_2)u_2$. To establish the required positions of the two vehicles cooperatively, three error metrics are required. The first two metrics pertain to each vehicle being at a prescribed radius r_d from the ground target. Additionally, in order to place the vehicles at 90 degrees of each other for the proper sensor fusion, the distances between the vehicles must be maintained as $r_d\sqrt{2}$. To satisfy the first two requirements, we wish to drive the following error to zero for each vehicle (e_1 and e_2):

$$e_1=(r_1^2-r_d^2)\quad\text{and}\quad e_2=(r_2^2-r_d^2) \tag{2}$$

For each individual vehicle, note the following error dynamics in terms of the system dynamics:

$$\dot{e}=2r\dot{r}-2r_d\dot{r}_d \tag{3}$$

$$\ddot{e}=2\dot{r}^2+2r\ddot{r}-2\dot{r}_d^2-2r_d\ddot{r}_d \tag{4}$$

Where $r=\sqrt{x^2+y^2}$, $\dot{r}=(x^2+y^2)^{-\frac{1}{2}}(x\dot{x}+y\dot{y})$, and the input *u* appears in,

$$2r\ddot{r}=-2r^{-2}(x\dot{x}+y\dot{y})^2+2(\dot{x}^2+\dot{y}^2)+2(\dot{x}y-x\dot{y})u \tag{5}$$

To satisfy the coupling requirement for the vehicles being a certain distance apart, we wish to drive the relative separation distance error to zero:

$$e_r=(x_2-x_1)^2+(y_2-y_1)^2-2r_d^2 \tag{6}$$

Note the following relative separation error dynamics:

$$\dot{e}_r=2(x_1-x_2)(\dot{x}_1-\dot{x}_2)+2(y_1-y_2)(\dot{y}_1-\dot{y}_2) \tag{7}$$

$$\ddot{e}_r=2(\dot{x}_1-\dot{x}_2)^2+2(x_1-x_2)(-\dot{y}_1u_1+\dot{y}_2u_2)+2(\dot{y}_1-\dot{y}_2)^2+2(y_1-y_2)(\dot{x}_1u_1-\dot{x}_2u_2) \tag{8}$$

Consider the following cooperative control Lyapunov function (CCLF) candidate:

$$V=\tfrac{1}{2}s_1^2+\tfrac{1}{2}s_2^2+\tfrac{k_r}{2}s_r^2 \tag{9}$$

where driving this value to zero will establish the following three invariant manifolds which in turn specify desired first order stable error dynamics regarding e_1, e_2, and e_r with a relative weighting of k_r: $s_1=\dot{e}_1+\lambda_1e_1$, $s_2=\dot{e}_2+\lambda_2e_2$, $s_r=\dot{e}_r+\lambda_re_r$. In order to drive the CCLF to zero, take the derivative with respect to time and then force it to be negative definite. Following the notation of Primbs et al [81,82,26]:

$$\dot{V}=V_{\mathbf{x}_1}^T\dot{\mathbf{x}}_1+V_{\mathbf{x}_2}^T\dot{\mathbf{x}}_2=V_{\mathbf{x}_1}^T\mathbf{f(x_1)}+V_{\mathbf{x}_1}^T\mathbf{b(x_1)}u_1+V_{\mathbf{x}_2}^T\mathbf{f(x_2)}+V_{\mathbf{x}_2}^T\mathbf{b(x_2)}u_2=s_1\dot{s}_1+s_2\dot{s}_2+k_rs_r\dot{s}_r \tag{10}$$

A substitution of appropriate relations given by the equations above results in:

$$\dot{V}=f_1+b_1u_1+f_2+b_2u_2+2f_r+b_{r1}u_1+b_{r2}u_2 \tag{11}$$

where f_1 and b_1 are functions *solely* of states regarding vehicle 1, and f_2 and b_2 are functions solely of states regarding vehicle 2:

$$f_1=s_1[2\dot{r}_1^2+2\lambda_1r_1\dot{r}_1-2r_1^{-2}(x_1\dot{x}_1+y_1\dot{y}_1)^2+2(\dot{x}_1^2+\dot{y}_1^2)] \tag{12}$$

$$b_1=2s_1(\dot{x}_1y_1-x_1\dot{y}_1) \tag{13}$$

$$f_2=s_2[2\dot{r}_2^2+2\lambda_2r_2\dot{r}_2-2r_2^{-2}(x_2\dot{x}_2+y_2\dot{y}_2)^2+2(\dot{x}_2^2+\dot{y}_2^2)] \tag{14}$$

$$b_2=2s_2(\dot{x}_2y_2-x_2\dot{y}_2) \tag{15}$$

Functions f_r, b_{r1} and b_{r2} contain the states of both vehicles and represent the coupling introduced between the vehicles by virtue of the desired error dynamics prescribed by invariant manifolds s_1, s_2 and s_r. These coupling functions are given by the following:

$$2f_r = k_r s_r[2(\dot{x}_1 - \dot{x}_2)^2 + 2(\dot{y}_1 - \dot{y}_2)^2 + \lambda_r \dot{e}_r] \quad (16)$$

$$b_{r1} = k_r s_r[2(x_1 - x_2)(-\dot{y}_1) + 2(y_1 - y_2)(\dot{x}_1)] \quad (17)$$

$$b_{r2} = k_r s_r[2(x_1 - x_2)(\dot{y}_2) + 2(y_1 - y_2)(-\dot{x}_2)] \quad (18)$$

Note that $f_1 \neq \mathbf{f}_1(\mathbf{x}_1)$, etc. The *f* and *b* notation has been adopted due to the fact that these functions express the dynamics of the overall *cooperative* system in a state space representation of the desired cooperative task. We now wish to ensure that $\dot{V}$ as given by Equation (11) is negative definite by dividing this task up in some kind of symmetric way between the two vehicles and in a way so that a vehicle can utilize information about the other vehicle (in the form of states, a combination of states, or a subset of states) in the same manner that the other vehicle (identical and doing the identical task) uses information about it. Likewise we could consider the asymmetric case where one vehicle has more responsibility for the task than the other, but this is not currently under consideration (although such considerations would be appropriate for heterogeneous teams of vehicles). We will assume that each vehicle has full state feedback of its own states. One sufficient condition for accomplishing this is to separate $\dot{V}$ into two symmetric contributions and then make each contribution negative definite. The sum of two negative definite functions is in turn a negative definite function. Consider the following separation of $\dot{V}$ into two symmetric contributions:

$$\dot{V} = \underbrace{f_1 + f_r + (b_1 + b_{r1})u_1}_{\substack{\text{Vehicle 1 responsible for enforcing} \\ \text{this portion negative definite}}} + \underbrace{f_2 + f_r + (b_2 + b_{r2})u_2}_{\substack{\text{Vehicle 2 responsible for enforcing} \\ \text{this portion negative definite}}} = \dot{V}_1 + \dot{V}_2 \quad (19a)$$

$$\dot{V}_1 = f_1 + f_r + (b_1 + b_{r1})u_1 \quad (19b)$$

$$\dot{V}_2 = f_2 + f_r + (b_2 + b_{r2})u_2 \quad (19c)$$

The division of $\dot{V}$ into these contributions constitutes the formation of the time derivative of the ACF's. In order for each vehicle to make it's contributed ACF rate negative definite, each needs information regarding the other vehicle as dictated by functions f_r, b_{r1} (for vehicle 1). Discussing this information sharing from the point of view of vehicle 1's responsibilities toward Equation (19a), it will need the following information about vehicle 2: θ_2, x_2, $\dot{x}_2$, y_2 and $\dot{y}_2$. An important point general to cooperative control among vehicles with knowledge about the other vehicles is the following: vehicle 1 has the ability to use it's knowledge of vehicle 2's dynamics in order to obtain $\dot{x}_2$ and $\dot{y}_2$ from θ_2, x_2 and y_2 by utilizing vehicle 2's known dynamic equations, Equations (1a). Therefore vehicle 1 can exploit the fact that it knows the dynamic behavior of vehicle 2 in order to construct an observer (in general this may also require knowledge of vehicle 2's control u_2). Once the required information is obtained, each vehicle can enforce the following relationships in order to make their respective contributions to $\dot{V}$ negative definite:

$$\text{Choose } u_1 \text{ to enforce}: f_1 + f_r + (b_1 + b_{r1})u_1 = -K_1|b_1 + b_{r1}|\sqrt{\tfrac{1}{2}s_1^2 + \tfrac{k_r}{4}s_r^2} \quad (20a)$$

$$\text{Choose } u_2 \text{ to enforce}: f_2 + f_r + (b_2 + b_{r2})u_2 = -K_2|b_2 + b_{r2}|\sqrt{\tfrac{1}{2}s_2^2 + \tfrac{k_r}{4}s_r^2} \quad (20b)$$

where the right hand side of Equations (20) are negative definite by choice of positive coefficients K_1 and K_2, selected to account for modeling error bounds in the typical sliding mode control sense. Also note (from the $\dot{V}$ term of Equation (19)) that each vehicle only acts based on its contribution to the positive definite scalar metric of the overall global error given by *V* (this must be the case given that u_1 cannot influence $\frac{1}{2}s_2^2$ and vice-versa). A typical run of this control law formulation is shown in Figure 5 (values used were: $v = 1$, $\lambda_1 = \lambda_2 = 2\pi/200$, $\lambda_r = 2\pi/50$, $k_r = 3$, $K_1 = K_2 = 0.3$, $r_d = 5$, with $u_1, u_2 \in [-1,1]$).

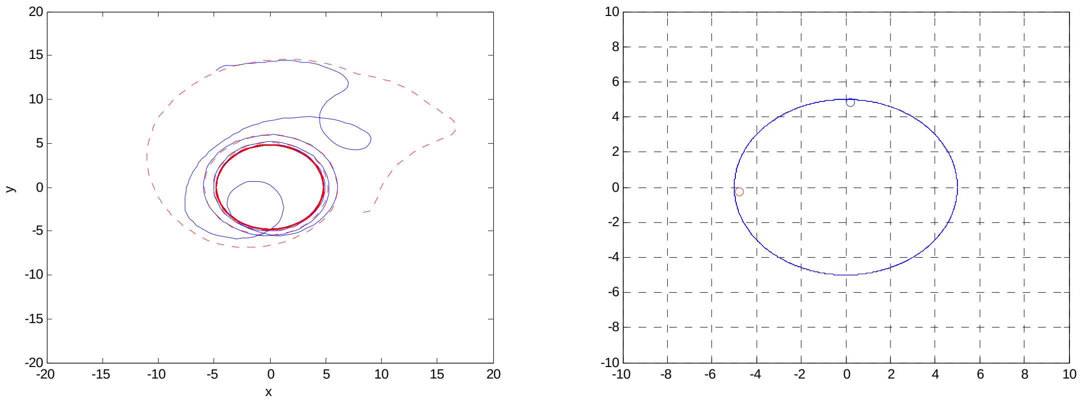

Figure 5: Control performance of the PF/CCLF/ACF approach in cooperatively establishing an orbit of two vehicles around a target (0,0) with a specified radial distance and a specified angular separation (relative distance). The plot on the left shows the trajectory of the two vehicles in establishing the cooperative orbit, and the plot on the right shows a snapshot of the desired relative angular separation of 90 degrees after the vehicles have converged to the desired cooperative orbit.

Several comments and discussion points can be made regarding the toy problem above. With regard to the hypothesis of unifying the idea of a potential field with the idea of a control Lyapunov function, the above example illustrates that the "potential" in this case is the CCLF of Equation (11). The manner in which this potential is "processed" by each vehicle is the manner in which each enforces Equations (20). This is similar to the physical case of say gravitational potentials specifying the force as the gradient of the potential. Although a gradient with respect to space is not explicitly employed here, the thinking is similar in that a scalar field is utilized to derive the control influence. There is however an important distinction between physics-based potentials and the "potential" used here; the global "potential" here as represented by Equation (9) is divided among the vehicles and in effect creates a separate potential for each to follow that is unique to an individual vehicle. This could be seen as an extension to the idea of deriving the "force" from the gradient of a potential field to something more general as represented by Equations (19). Viewed from a controls standpoint, the control law seeks to place the system on the invariant manifolds s_1, s_2 and s_r. Once on the manifold, the error converges to zero as specified by the desired error dynamics. An important piece of the problem is however missing in this toy problem; the invariant manifolds s_1, s_2 and s_r may not represent achievable trajectories in the error space. This is due to two reasons for this specific case. First, the non-holonomic dynamics are not accounted for in specifying achievable error dynamics. This problem has been addressed extensively by Kyriakopoulos et. al. [51], and others in the literature. The second inadequacy/inaccuracy of the above example is that the control constraints have not been explicitly accounted for (turning rate limits). Papers by Wu and Jayasuriya [104] and others have addressed this concern. In particular, Åström and Furuta [2] have shown that an inverted pendulum can by swung up under hard input saturation constraints by controlling the energy of the pendulum. This hints at the fact that given the right metric, or PF in our case, even input constraints can be explicitly accommodated. Ogren and Leonard [72] address similar concerns in their formulation of a navigation function. Further evidence that the example problem has not been cast in a strictly proper sense can be observed from Figure 5b by the fact that although the system came close to the desired specifications, exact tracking was not achieved (desired radius and phase both contain steady state error).

The most important point to take away from the simple example problem presented above is that a method is proposed for the *cooperative* control of multiple dynamic systems that explicitly incorporates interaction among vehicles. The idea behind the proposed method elucidates the notion of defining an "agent" as a symmetric contribution to decreasing the CCLF in a manner that transforms the problem from a traditional centralized multi-input, multi-output control formulation into one which distributes the

control task among physical units. The outline of the method followed in the example problem also sheds light on to the information required to be shared among vehicles. The preliminary method additionally sheds light on the notion of defining potential fields in a more general sense than those inspired by purely physics-based potential fields. The problem above, for example, can implicitly incorporate collision avoidance between the vehicles by enforcing a relative separation distance. As presented above, this was not strictly the case due to the way the CCLF was split between vehicles; each vehicle was responsible for decreasing the *weighted* penalty of being far away from the desired radius *and* of being at a distance other than the desired relative separation distance. Due to this weighted penalty, it was not enforced that each vehicle monotonically decrease its relative separation distance error – but it could have been. Such concerns shed further light on how to split and enforce the contributions of the CCLF among agents. Lastly, the proposed approach, if cast correctly to take into account the deficiencies noted above, is based in stability theory and can therefore offer rigorous analysis tools concerning performance and robustness of both the system as a whole, and the vehicles or agents individually.

However, these points being stated, many unanswered and open questions remain. Exactly how can the CCLF and PF be defined to explicitly incorporate or address dynamic, state and input constraints? Are their better candidate CCLF's that would reduce the required amount of shared information, and how would such CCLF's be constructed in general to achieve this (since ideally each vehicle would only need to know the current scalar value of the CCLF and its own states)? How can this preliminary approach be cast to design some sense of optimality by linking the CCLF, which through the observations of Primbs [81,82] is a scaling of some value function corresponding to some unknown cost function, to properties of its cost function. What happens when this CCLF/PF/ACF method is applied to a general number of vehicles? What happens in terms of robustness when team vehicles are added or subtracted as the task is progressing, and how would this be accounted for in the structure of the approach? If a state observer is employed by a vehicle to estimate the states of another vehicle, how does this need to be incorporated into the overall structure to ensure stability and robustness? These and other questions will be answered.

5. ADDITIONAL DOCUMENTATION

Several simulation files have been generated by this work. Below is a list of the most relevant and finalized simulations.

- `coop_orbit6.mdl` : a stand_alone simulation that shows the robustness of assigning two vehicles to a CGMTE sensor fusion scenario. A manual switch is located with the "target" block to change from a smooth target track to one with sudden 90 degree turns.
- `coop_orbit6_plot.m` : shows an animation and is meant to be run directly after coop_orbit6.mdl is executed.
- `load CGMTE_run_xx.mat`: nine data files of the assignment of 5 UAV's to 3 targets in a CGMTE scenario. Generated by CGMTE_assignment which calls a number of files.
- `CGMTE_plot.m` : shows an animation of the data in CGMTE_run_xx.mat.

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